A brief outline of my current and intended research

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Currently I am most interested in the development of a combinatorial approach to the representation theory. I have written and put on my homepage, a few articles describing the invariants and decompositions of tensor products of polynomial representations of $SL(2)$ in the terms of outerplanar graphs, i.e. graphs, edges of which can be drawn in the upper half-plane without intersections. The analogous description can be done for invariants and representations of $SL(n)$, and I am including that in my thesis.

Also, in my thesis I give the description of polynomial representations of any semi-simple Lie groups and algebras and Kac-Moody algebras in terms of walks on the corresponding weight lattices. This theory can be considered as a discrete analog of Littelmann's path theory, and it has very interesting connections with Kashiwara's crystals and Gelfand-Zetlin diagrams as well (and it gives different pictures and formulas for $SL(n)$, than both Gelfand-Zetlin and Young diagram theories). I would like to extend this to q-deformations of Kac-Moody algebras.

I use quite a different combinatorial approach to the description of the representations of unipotent groups of Lie type, through 'diagrams of representations'. This approach leads to various applications, including explicit formulas for fractional residues (i.e. invariants of some generalizations of differential forms), and the generalization of Grothendieck's residue.

Conversely, these three combinatorial approaches to representations allow us to use known representation-theoretical results to get the explicit formulas for the enumeration of the corresponding combinatorial objects, like walks on lattices, or the counting of some specific graphs.

Another topic of my research, of which I have more questions than answers; is total positivity and Polya frequencity of series of representations. This theory has very promising (possible) applications to the number theory, especially to the Riemann hypothesis and its generalization to L-series.