Math 240,
the course of Dr. Mihailovs

Midterm 1

July 9, 1998

Name ______________________________

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1. Let $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$. Find $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$.

2. Let $A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Find AB and B^{-1}.

3. Find the equation of the plane through $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ perpendicular to the line $x = 3 + 2t,\ y = 1 - t,\ z = t$.

4. Let $\mathbf{a} = \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$. Find a unit vector parallel to $\mathbf{b}$ and the projection of $\mathbf{a}$ in the direction of $\mathbf{b}$.

5. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 6 \end{pmatrix}$. Write out the Laplace expansion of $\det A$ by column 2 (do not compute the determinant).

6. Compute $(\mathbf{e}_1 + 2\mathbf{e}_2) \wedge (3\mathbf{e}_1 + 4\mathbf{e}_2)$.

7. Let T_A be a linear mapping $T_A : x \mapsto Ax$ where $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. Sketch the image under T_A of the unit square $\mathbf{D}$ spanned by $\mathbf{e}_1$, $\mathbf{e}_2$ and oriented counterclockwise. Does T_A preserve or reverse the orientation?

8. Find a parametric equation for the line through $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$.

9. Is the line $x+y=-1$ a subspace of $\mathbb{R}^2$? Is the half-space $z \geq 0$ a subspace of $\mathbb{R}^3$?

10. Find the eigenvalues and corresponding eigenvectors for the symmetric matrix $A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

11. With A as above, find an orthogonal matrix B such that $B^{-1}AB$ is diagonal.

12. Compute the area of the parallelogram spanned by $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ (same as in Problem 1).

13. In each case below determine whether the given vectors are linearly dependent or linearly independent (don't compute):

(i) $\binom{1}{-1}$, $\binom{2}{3}$, $\binom{-2}{1}$.

(ii) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$.

14. Suppose that one can reduce the augmented coefficient matrix of the system $A\mathbf{x} = \mathbf{0}$ to the eschelon form $\begin{pmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$. What is the dimension of the space of solutions of the given system?

15. With regard to the usual basis $\mathbf{e}_1 = \binom{1}{0}$, $\mathbf{e}_2 = \binom{0}{1}$ in $\mathbb{R}^2$, P has coordinates $\binom{2}{1}$. What are the coordinates of P with regard to a new basis $\mathbf{b}_1 = \binom{1}{1}$, $\mathbf{b}_2 = \binom{-1}{1}$? (Hint: use Problem 2.)

16. Suppose that A is a 2×2 matrix and that the following sequence of basic operations reduces A to I:

$$A \xrightarrow[\frac{1}{2}\text{ row 1}]{\text{row 1}} A_1 \xrightarrow[\text{row 2}-\text{row 1}]{\text{row 2}} I$$

Find the elementary matrices which produce these operations. Find A^{-1}.

17. Let $T_A : \mathbb{R}^4 \to \mathbb{R}^3, \quad \mathbf{x} \mapsto A\mathbf{x}$ where $A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Find a basis for the range of T_A and give its dimension. Find the dimension of the null space of T_A.

18. Let $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Write down e^{At} (just give the answer.)

19. With $C = e^{At}$ as above, what does the map $\mathbf{x} \mapsto C\mathbf{x}$ do to $\mathbb{R}^2$?