Math 240,
the course of Dr. Mihailovs

Midterm 2

July 23, 1998

Name ______________________________

Page	1	2	3	4	5	6	7	8	9	10	Total
Points											

1. Let $\mathbf{F} = \begin{pmatrix} xy \\ yz \\ zx \end{pmatrix}$. Find $\nabla\cdot\, \mathbf{F}$ and $\nabla\times\, \mathbf{F}$.

2. Compute $\int_D 6x^2 y dx dy$ where D is a unit square $0 \leq x \leq 1, 0 \leq y \leq 1$.

3. Find the rate of increase per unit distance of the function $f = xy^2 z$ at $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ in the direction of $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$.

4. Let $\mathbf{C}$ be the half-circle $x^2 + y^2 = 1, \ x \geq 0$ oriented counter-clockwise. Compute $\int_{\mathbf{C}} ydx - xdy$.

5. Let $\mathbf{C}$ be the boundary of unit square $0 \leq x \leq 1, 0 \leq y \leq 1$, oriented counter-clockwise. Use Green's Theorem to evaluate $\int_{\mathbf{C}} 2ydx + 3xdy$.

6. Let $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$. Write down A^{-1}. Let $B = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 2 & 1 \\ -3 & 1 & -4 \end{pmatrix}$. Write down the Laplace expansion of $\det B$ by the first column. Do not evaluate it.

7. Let D be the quater of the unit disc $x^2 + y^2 \leq 1$ which lies in the first quadrant, $x \geq 0, y \geq 0$. Use polar coordinates to evaluate $\int_D x dx dy$.

8. V is the upper half of the unit ball $x^2 + y^2 + z^2 \leq 1,\ z \geq 0$. Use spherical polars to evaluate $\int_V \sqrt{x^2 + y^2 + z^2} dxdydz$.

9. Compute the volume of the solid body bounded by $z = 2 - x^2 - y^2$ and the plane $z = 0$.

10. Let $T_A : \mathbb{R}^4 \to \mathbb{R}^3, \quad \mathbf{x} \mapsto A\mathbf{x}$ where $A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$. Find a basis for the range of T_A and a basis for the null space of T_A.

11. S is the surface $z = 1 - x^2 - y^2,\ z \geq 0$. D is the unit disc $x^2 + y^2 \leq 1$. dA is the surface element on S. Write $\int_S (5 - 4z) dA$ as an integral over D. Do not evaluate.

12. S is the surface of the unit cube $0 \leq x \leq 1,\ 0 \leq y \leq 1,\ 0 \leq z \leq 1$. $\mathbf{n}$ is the outward drawn unit normal on S. Use the divergence Theorem to evaluate $\int_S \mathbf{F} \cdot \mathbf{n} dA$ where $\mathbf{F} = \begin{pmatrix} 2x \\ 3y \\ -4z \end{pmatrix}$.

13. Write out a careful statement of Stokes' Theorem (use a diagram).

14. Let $\mathbf{D}$ be the unit square $0 \leq x \leq 1,\ 0 \leq y \leq 1$ oriented by taking the boundary clockwise. Evaluate $\int_{\mathbf{D}} x dy \wedge dx$.

15. Suppose that A is a 2×2 matrix and that the following sequence of basic operations reduces A to I:

$$A \xrightarrow[\frac{1}{2}\text{ row } 2]{\text{row } 2} A_1 \xrightarrow[\text{row } 2 - \text{row } 1]{\text{row } 2} I$$

Find the elementary matrices which produce these operations. Find A^{-1}.

16. Let $Q = x_1^2 - 4x_1x_2 + x_2^2$. Determine whether the origin is a max, min, saddle point, or none of these.

17. Let $\mathbf{C}$ be a path in $\mathbb{R}^3$ running from the origin to the point $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

Detrmine whether $I = \int_{\mathbf{C}} xdx + ydy + zdz$ depends on the path or is the same for all paths from $\mathbf{0}$ to $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

18. Let $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Write down the general real solution to $\dot{x} = Ax$.

19. Evaluate $\int_S 2x^2 + 3y^2 + 4z^2 \; dA$ where S is the unit sphere $x^2 + y^2 + z^2 = 1$ and dA is the surface element.