Past Research

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My past research interests were mainly in Representation Theory, Invariant Theory, and Combinatorics.

Combinatorics

- I found a simple proof of the convolution formula for log-concave sequences, see [1]. It can be easily generalized for the proof of the convolution formula for the Polya frequency sequences and totally positive series (unpublished).
- I found the series of formulas for the numbers of lucky tickets and Petrovsky numbers as well as their asymptotics. See [2, 3].
- I found the formulas for the counting of random walks on the Schur operations on (weighted) graphs. See [4, 5].
- I introduced 4 series of graphs, linear n-wave graphs, symplectic $2n$-wave graphs, orthogonal $(2n + 1)$-wave graphs, and G_2-wave graphs and found explicit formulas for the enumeration of them, using my formulas for the counting of random walks

on the Schur operations on graphs. See [4, 5, 6, 7, 8]. I'll talk about that at the Joint Mathematics Meetings in Baltimore.

Invariant Theory

- I found explicit formulas for the invariants and normal forms of the coadjoint representation of the (infinite-dimensional) group of automorphisms of a line using my Diagram Method. See [4, 9, 10, 11].

- I found explicit formulas for the fractional residues, i. e. invariants of generalized differential forms $\psi(t)(dt)^{\lambda}$ on a line and the corresponding normal forms using my Diagram Method. See [4, 9, 10].

- I found explicit formulas for bases of $SL(n)$-invariants in $V^{\otimes m}$ where V is the n-dimensional linear space with the standard action of $SL(n)$. I parametrized these bases by linear n-wave graphs with m vertices. See [4, 7].

- I found explicit formulas for bases of $Sp(2n)$-invariants in $V^{\otimes m}$ where V is the $2n$-dimensional linear space of the defining representation of $Sp(2n)$. I parametrized these bases by symplectic $2n$-wave graphs with m vertices and found explicit formulas for the dimensions using my formulas for the counting of walks on Schur operations on lattices. See [4, 5, 6].

- Recently I found explicit formulas for $SO(2n+1)$ and G_2-tensor invariants and dimensions of the invariant spaces. I'll talk about that at the Joint Mathematics Meetings in Baltimore.

- I wrote a series of Maple programs for binary tensor invariants and outerplanar graphs, see [12].

Representation Theory

- I developed the Diagram Method for the studying of representations of unipotent Lie groups and nilpotent Lie algebras. See [4, 9].
- I used my Diagram Method to completely describe the unitary representations of the group of unipotent real $n \times n$ upper-triangular matrices for $n \leq 9$ and to construct some counterexamples in a general case. See [4, 9, 13]
- I found the explicit formulas for bases of the decomposition of $V^{\otimes m}$ into a sum of irreducible representations of $SL(2)$, where V is a 2-dimensional linear space with the standard action of $SL(2)$. See [4, 14].
- I found a way to apply the Kirillov's Orbit Method to finite groups. I presented a lecture on that at the Joint AMS/MAA Meetings in Washington, DC. See [15].

References

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