Publication List

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1 Books

1.1. Alec Mihailovs. *A Combinatorial Approach to Representations of Lie Groups and Algebras*, Springer-Verlag New York (2002).

352 pages , 6 1/8 x 9 , hardcover, ISBN: 0-8176-4251-X.

Presents the first detailed exposition of wave graphs, diagrams of representations, and random walks on lattices, with applications to representation theory and invariant theory. Apart from new results and refreshing ideas, interesting unsolved problems are presented. For researchers and grad students in representation theory, Lie theory, combinatorics, invariant theory, and quantum mechanics.

2 Papers published

2.1. Alec Mihailovs. On the log-concavity, *Kvant* 11/12 (1993), p. 1-9 (in Russian).

3 Papers accepted

3.1. Alec Mihailovs. Tensor decompositions for $SL(2)$ and outerplanar graphs, math.RT/9712259 (1997). Accepted for publication by *Journal of Combinatorial Theory*, Series A.

21 pages. The main result of this article is the decomposition of tensor products of representations of $SL(2)$ in the sum of irreducible representations parameterized by outerplanar graphs. An outerplanar graph is a graph with vertices $0, 1, \ldots, m$, edges of which can be drawn in the upper half-plane without intersections. I allow for a graph to have multiple edges, but don't allow loops.

4 ArXive preprints

4.1. Alec Mihailovs. Tensor invariants of $SL(n)$, wave graphs and L-tris, math.RT/ 9802119 (1998).

8 pages. The space of invariants of a tensor product of representations of $SL(n)$ is provided with the basis parameterized by wave graphs introduced here especially for this purpose. The proof utilizes a game similar to Tetris, named here L-tris.

4.2. Alec Mihailovs. Fractional residues, math.RT/9803018 (1998).

17 pages. Invariants of generalized tensor fields on a line are classified using special polynomials $P_{mk}^{(-1/l)}$ introduced here for this purpose. For the case of positive characteristic, a new invariant of formal power series, a width, is defined. Some applications to the geometric quantization of a line and conformal quantum field theory are discussed as well.

4.3. Alec Mihailovs. Diagrams of representations, math.RT/9803079 (1998).

19 pages. For a representation of a Lie algebra, one can construct a diagram of the representation, i. e. a directed graph with edges labeled by matrix elements of the representation. This article explains how to use these diagrams to describe normal forms, orbits and invariants of the representation, especially for the case of nilpotent Lie algebras.

4.4. Alec Mihailovs. Symplectic tensor invariants, wave graphs and S-tris, math.RT/ 9803102 (1998).

16 pages. The spaces of invariants of tensor powers of the defining representation of $Sp(2n)$ are provided with the bases parameterized by symplectic wave graphs introduced here especially for this purpose. The proof utilizes a game similar to Tetris, named here S-tris. This work continues my previous work [4.1] on the tensor invariants of $SL(n)$, wave graphs and L-tris.

4.5. Alec Mihailovs. Enumeration of walks on lattices. I, math.CO/9803128 (1998).

37 pages. This work develops a methodical approach to the counting of walks on Cartesian products, biproducts, symmetric and exterior powers and bipowers, Schur operations, coverings and semicoverings of weighted graphs. For weight and root lattices of semisimple Lie algebras, this approach allows us to compute various combinatorial and representation-theoretical constants, in particular, the number of plane symplectic wave graphs with given number of vertices.

4.6. ALEC MIHAILOVS. The Orbit Method for Finite Groups of Nilpotency Class Two of Odd Order, math.RT/0001092 (2000).

16 pages. First, I construct an isomorphism between the categories of (topological) groups of nilpotency class 2 with 2-divisible center and (topological) Lie rings of nilpotency class 2 with 2-divisible center. That isomorphism allows us to construct adjoint and coadjoint representations as usual. For a finite group G of nilpotency class 2 of odd order, I construct a basis in its group algebra $\mathbb{C}[G]$, parameterized by elements of $\mathfrak{g}^*$ so that the elements of coadjoint orbits form bases of simple two-side ideals of $\mathbb{C}[G]$. That construction gives us a one-to-one correspondence between G-orbits in $\mathfrak{g}^*$ and classes of equivalence of irreducible unitary representations of G, implying a very simple character formula. The properties of that correspondence are similar to the properties of the analogous correspondence given by Kirillov's orbit method for nilpotent connected and simply connected Lie groups. The diagram method introduced in my article [4.3] and my thesis [5.3], gives us a convenient way to study normal forms on the orbits and corresponding representations.

5 Theses

5.1. ALEC MIHAILOVS. *Lucky Tickets and the Petrovsky Numbers*. Bachelors thesis, University of Latvia, 1995.

50 pages (in Russian).

5.2. ALEC MIHAILOVS. *The Petrovsky Numbers and Multiplicities of Representations*. Masters thesis, University of Latvia, 1995.

50 pages (in Russian).

5.3. ALEC MIHAILOVS. *A Combinatorial Approach to Representations of Lie Groups and Algebras*. Ph.D. thesis, University of Pennsylvania, 1998.

134+vii pages. First I describe the invariants and decompositions of tensor products of polynomial representations of $SL(2)$ in the terms of outerplanar graphs, i.e. graphs with the vertices $0, 1, \ldots, m$, the edges of which can be drawn in the upper half-plane without intersections. Then I use wave graphs introduced here to give an analogous description for tensor invariants of $SL(n)$ and $Sp(2n)$. I use quite a different combinatorial approach to the description of the representations of unipotent groups of Lie type, through 'diagrams of representations'. This approach leads to various applications, including explicit formulas for fractional residues (i.e. invariants of some generalizations of differential forms). Conversely, these combinatorial approaches to representations allow us to use known representation-theoretical results to get the explicit formulas for the enumeration of the corresponding combinatorial objects, like walks on lattices, or the counting of some specific graphs.

6 Online

6.1. Alec Mihailovs. Weights and roots. Preprint (1997).

5 pages. A survey of the basic constructions used to classify and study representations and invariants of semisimple Lie groups and algebras. Accessible from http://www.mihailovs.com/Alec/papers.html.

6.2. Alec Mihailovs. Tensor invariants of $SL(2)$ and outerplanar graphs. Preprint (1997).

7 pages. The space of invariants of a tensor product of representations of SL(2) is provided with the basis parameterized by outerplanar graphs. Accessible from http://www.mihailovs.com/Alec/papers.html.

6.3. Alec Mihailovs. Maple Programs for Binary Tensor Invariants and Outerplanar Graphs. Preprint (2002).

68 pages. Accessible from http://webpages.shepherd.edu/amihailo/maple.asp.

6.4. Alec Mihailovs, Mike May, S. J. *Abstract Algebra.* Maple PowerTool (2002).

Accessible from http://www.mapleapps.com/powertools/abstractalgebra/abstract algebra.shtml and from http://webpages.shepherd.edu/amihailo/maple.asp.

6.5. Alec Mihailovs. *Problems of the Week* (2001-2002).

17 problems with solutions. Accessible from http://www.shepherd.edu/mathweb/problems.html.

6.6. Alec Mihailovs. WWW sites `http://www.mihailovs.com/Alec`, `http://webpages.shepherd.edu/amihailo` and others (1996–2002).

7 Talks

7.1. ALEC MIHAILOVS. Wave graph bases of tensor invariants of classical Lie groups and algebras (October 1, 1998). Was presented at the Joint AMS/MAA Meetings in San Antonio, Texas, January 13–16, 1999 (January 16, 9:15 a.m., AMS Session on Topological Groups and Lie Groups.)

> Introduced in author's thesis [5.3], wave graphs give us a new combinatorial approach to the representations and invariants of classical Lie groups and algebras. For example, wave graphs
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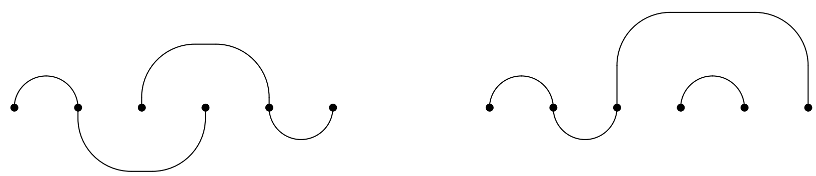
> 
>
> correspond to the following invariants of $SL(3)$ and $Sp(4)$:
>
> $$((x \wedge y \wedge z) \otimes (x \wedge y \wedge z))^{(34)}, \quad ((\omega \wedge \omega) \otimes \omega)^{(465)}$$
>
> where $\omega = p_1 \wedge q_1 + p_2 \wedge q_2$.

7.2. ALEC MIHAILOVS. The Orbit Method for Finite Groups . Was presented at the Joint AMS/MAA Meetings in Washington, DC, January 19–22, 2000 (January 19, 5:30 pm, AMS Session on Group Theory.)

> For a finite nilpotent group G, I define (by an inductive procedure) a finite G-module $\mathfrak{g}^*$ such that there is a one-to-one correspondence between G-orbits in $\mathfrak{g}^*$ and classes of equivalence of irreducible unitary representations of G. The properties of that correspondence are similar to the properties of the analogous correspondence given by Kirillov's orbit method. For instance, if G is the group of the upper-triangular unipotent $n \times n$ matrices with coefficients from a finite commutative ring A, one can choose the additive group of lower-triangular nilpotent $n \times n$ A-matrices as $\mathfrak{g}^*$, defining the action of G in $\mathfrak{g}^*$ the same way as Kirillov did in his classical article. Using diagram method introduced in my thesis [5.3], one can classify normal forms on the orbits and corresponding representations. I discuss the generalization of that construction for other classes of groups as well. In particular, for symmetric groups all the theory looks completely different from the standard approach.

7.3. ALEC MIHAILOVS. Wave graph bases of tensor invariants. A talk at Duke's Algebraic Geometry Seminar on February 12, 2002.

31 slides are available from http://webpages. shepherd.edu/amihailo/presentation.htm. The abstract is at http://www.math.duke.edu/mcal?abstract-4196.The description of tensor invariants using wave graphs is a matter of increasing interest. Although the classic theory and its applications to quantum electrodynamics, knot theory, and related fields, are well established, the wave graphs introduced in the presenter's thesis and their applications to the topology of the moduli spaces of curves and the structure of the mapping class groups of surfaces were discovered only very recently.

7.4. Alec Mihailovs. Wave graph bases of tensor invariants of $SO(2n+1)$ and G_2, scheduled on Thursday, January 16, 2003, 11:15 am, in the AMS Session on Group Theory at the Joint Mathematics Meetings in Baltimore.

The bases of $SL(2)$-invariants of tensor powers of the two-dimensional vector space with the standard action of $SL(2)$ were described using non-intersecting arcs by Rumer, Teller, and Weyl in 1932. In 1991 Furlan, Stanev and Todorov found the generalization of that construction to the description of $SL(2)$-invariants of tensor powers of the other irreducible finite-dimensional representations of $SL(2)$. In his Ph.D. thesis [5.3] in 1998, Alec Mihailovs described the bases of $SL(n)$-invariants and $Sp(2n)$-invariants of tensor powers of the standard representations of $SL(n)$ and $Sp(2n)$ using wave graphs generalizing the systems of non-intersecting arcs. In 1998 and 2001, Shigeyuki Morita found applications of wave graph bases to the topology of the moduli spaces of curves, to the structure of the mapping class groups of surfaces, and to Faber's conjecture. Recently Alec Mihailovs found the wave graph description of the bases of $SO(2n+1)$-invariants and G_2 invariants.